Rain-Induced Vibration

R. Rodeman* and D. B. Longcope*
Sandia National Laboratories, Albuquerque, New Mexico

The purely longitudinal response of an elastic rod structural model of a space vehicle moving through rain at a constant velocity has been calculated. A statistical model and a related deterministic model which describe the rain-induced force on the nose of the space vehicle are formulated. These models are used to predict first- and second-order statistical properties of the excitation and response. Results show that the average force due to the raindrops in a typical weather encounter is small compared to the atmospheric drag. It is further found that the structural response is dominated by frequencies in the 1 MHz regime. A comparison between the responses of the rod theory which allows only longitudinal motion and that which includes the effects of radial inertia and radial shear suggests that the simpler theory provides an upper bound on the axial response of a rod cross section.

Nomenclature

a = spherical raindrop radius
a_f = radius of cylindrical rod
$\bar{a}$ = deterministic model raindrop radius
A = random variable denoting raindrop radius
A_f = cross-sectional area of rod
c, c_D, c_s = bar, dilatational, and shear wave speed
C_n = nth Fourier coefficient
$\hat{C}$ = time-averaged estimate of force covariance
E = expectation operator and Young's modulus
f_A = probability density function for raindrop radius
F_t = total force on cross-sectional area
$\bar{F}$ = time-averaged estimate of the mean value of the force
$h, \hat{h}$ = incremental force pulse produced by the impact of a hypervelocity raindrop, Fourier transform of h
H = Heaviside function
I = impulse imparted by raindrop impact
k = momentum multiplication factor
K, K_1 = correction factors in Mindlin-Herrmann theory
K_F = autocovariance function of the excitation
M = mass of raindrop
$\dot{M}$ = mass encounter rate
N = number density of raindrops per size per unit volume
N_0 = empirically determined constant characterizing specific storms
N_t = number of raindrops encountered during $t-t_0$
P = representative maximum pressure
P_H = Hugoniot pressure
P^* = kinetic pressure
$\hat{R}$ = time-averaged estimate of the autocorrelation function
S_A = covariance spectral density of induced acceleration
$\bar{S}_A, \bar{S}_{A_2}$ = nondimensional covariance spectral density of induced acceleration for force models 1 and 2
S_F = covariance spectral density of the excitation
$\bar{S}_F, \bar{S}_{F_2}$ = nondimensional spectral density of the excitation for force models 1 and 2
t, t_0 = time and initial time
u = axial displacement
V = velocity
w = radial displacement
x = axial coordinate
α = empirically determined cloud parameter which is also the reciprocal of average raindrop radius
ΔT = separation time between impacts in deterministic model
λ = raindrop average encounter rate
λ_D = raindrop average encounter rate in deterministic model
λ_l, μ = Lame parameter
$\rho, \rho_A, \rho_L, \rho_W$ = bar, atmosphere, cloud, and water mass density
σ_A, σ_F = standard deviation of acceleration and excitation
$\hat{\sigma}$ = time-averaged estimate of standard deviation of the excitation
τ_n = time of impact of the nth raindrop
Φ_F = characteristic functional of the excitation
ω, ω' = circular and characteristic frequencies
Ω = nondimensional frequency

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*Staff Member, Analytical Mechanics Division IV.

Introduction

WEATHER effects on space vehicles have been a subject of investigation for over 20 years. The primary concern has been focused on the erosive effect of the weather encounter. More recently weather-induced vibration has become a concern. Instrumented flights through weather have indicated that it does indeed produce high-level vibration. Unfortunately, a complete, valid measurement of both the weather and the induced vibration has not been obtained. However, available information shows that the weather-induced vibration environment at certain vehicle locations greatly exceeds levels experienced in clear-air flights.

A theoretical distribution for the size of raindrops as a function of rain intensity was originally obtained by Marshall and Palmer.[1] That distribution has prevailed and today meteorologists are in apparent agreement as to the form of the model used to describe the number density of raindrops per size per unit volume of the atmosphere.

Strike and Lasker[2] first studied responses for a vehicle moving through weather. In support of their effort, high-velocity, single simulated raindrop impact tests were performed as reported in Ref. 2.

In the present study we consider that as a vehicle moves at constant velocity through a rain cloud it will encounter some large, but finite, number of raindrops. The number and size that actually impact the vehicle are dependent upon the specific storm, the vehicle, and its velocity. For the class of vehicles in which we are interested only a small fraction of the particles will be slowed or stopped by the bow shock layer and all shock-layer effects are therefore ignored. Each of these particles will impart momentum to the vehicle. Although the number of particles encountered, the size of each, and the times and places of impact cannot be determined in advance, the overall force imparted to the vehicle can be described in a statistical sense. Also, as reported in Ref. 2, since normal impacts impart an order of magnitude more momentum than oblique impacts, the nose of the space vehicle is assumed to be a flat area and impacts aft of it are ignored.

The force imparted to the assumed flat nose of the space vehicle is modeled as a deterministic model and as a statistical model. In the deterministic model the total force on the vehicle is represented as a superposition of forces due to fixed-size raindrops arriving at a constant rate. Time-averaged properties of this model are obtained. For the statistical model the total force is represented by a filtered Poisson process, with the force due to a single encounter taken from Ref. 2. Following Ref. 3 a characteristic functional is obtained from which first- and second-order statistical properties of the loading are derived. A comparison of the results of the two models is made. It is found that the average force due to the weather encounter is small compared to atmospheric drag. The covariance spectral density of the excitation is constant until the frequency increases to a value comparable to a characteristic frequency dependent on both the average raindrop radius and the vehicle velocity, at which point the level begins to attenuate. The constant level of the covariance spectral density increases as the third power of both the average raindrop radius and the vehicle velocity. The two models predict the same average force on the vehicle due to the weather encounter. It is found that the average force is independent of raindrop size and the only weather parameter that it is dependent upon is the water density of the cloud system. The covariance spectral densities of the two models may be made to agree closely by a suitable choice of the fixed-size raindrop in the deterministic model. In addition, it is found that the coefficients of variation for the two models are quite similar for typical values of the problem parameters.

Although a variety of structural models might be used, the space vehicle is modeled as a one-dimensional homogeneous elastic rod of circular cross section with an excitation at one end. For this model, the response is purely longitudinal and is independent of radial location of the impact. The structural acceleration response is dominated by high frequencies on the order of 1 MHz. A comparison made between the responses predicted by the elementary rod theory and the Mindlin-Herrmann theory[4] which treats radial motion and radial shear suggests that the elementary theory may provide an upper bound on the average response of a cross section of the rod. How close an upper bound it represents is uncertain.

Formulation of the Problem

Weather Model

Small and large ice crystals, snow, and rain are present at different altitudes in a typical cloud system. These particles exist in many different shapes and sizes. For our model all of the particles are considered to be spherical water droplets with diameters equal to the equivalent diameter of the melted particle. The number density of rain drops per size per unit cloud volume is taken from Ref. 3 as

$$N(a,r)=N_0(r)e^{-\alpha(r)a}, \quad 0\le a<\infty \tag{1}$$

where N is the number of particles per radius a per unit cloud volume, r a precipitation rate, and α and N_0 empirically determined constants characterizing specific storms. Particles with diameters as large as 1 mm occur in number densities in tens of particles/m^3 while particles with diameters of 100 μm occur in tens of thousands/m^3.

The probability density function for the raindrop radius is derived from Eq. (1) as

$$f_A(a)=\alpha e^{-\alpha a}, \quad 0\le a\le\infty \tag{2}$$

where A is the random variable denoting the raindrop radius. From Eq. (2) it may be seen that the average and standard deviation of the raindrop radius is

$$E(A)=\sigma_A=1/\alpha \tag{3}$$

where E is the expectation operator, $E(A)$ the average value of A, and σ_A the standard deviation of A. The raindrop average encounter rate is obtained by multiplying the cloud volume swept per unit time by the nose of the space vehicle and the total number of raindrops per unit volume

$$\lambda=A_f VN_0/\alpha \tag{4}$$

where A_f is the frontal area of the vehicle and V its velocity.

The mass encounter rate $\dot{M}$ for the vehicle is given as

$$\dot{M}=\rho_L A_f V \tag{5}$$

where ρ_L the cloud density is obtained from

$$\rho_L=\int_0^\infty \frac{4}{3}\pi\rho_w a^3 N_0 e^{-\alpha a}\mathrm{d}a \tag{6}$$

and ρ_w is the mass density of water.

Single-Particle Impact

As in Ref. 2, the force pulse produced by the impact of a hypervelocity drop with the nose of a space vehicle is assumed to be in the form

$$h(t)=\pi a^2 p\exp(-\pi a^2 pt/I), \quad t\ge 0$$
$$=0, \quad t<0 \tag{7}$$

where p is a representative maximum pressure and I the impulse imparted to the nose by the drop impact. The impulse is given by

$$I=kmV \tag{8}$$

where m is the mass of the drop, V the relative velocity between the drop and the space vehicle, and k a momentum multiplication factor.

When a hypervelocity particle impacts a solid body, the momentum imparted to the body can be greater than the momentum of the particle as a result of ejecta from the particle or body and $k\ge 1$ accounts for this. Experiments reported in Ref. 2 showed $k\simeq 1.3$ for glass spheres of 1 mm diam normally impacting various carbon materials at 5000 m/s. The maximum force is $\pi a^2 p$ and the representative impact pressure p may be adjusted to reflect nonlinear dissipation effects near the impact site in the solid body. The maximum reasonable value of p is p_H, the Hugoniot pressure generated just after the normal impact at the drop/space vehicle relative velocity of two flat plates, one of water and the other of the nose material. For a given impulse the Hugoniot pressure gives the force pulse with the shortest duration and highest frequency content. Smaller values of p will give longer duration and lower frequency content force pulses and thereby approximate the dissipation effects at axial locations in the nose which are several drop radii or more from the impact site.

Multiple Particle Impacts

The force as a function of time F_t; $t \geq 0$ on the flat nose of the vehicle moving at constant velocity through a rain cloud is modeled as

$$F_t = 0, \qquad N_t = 0$$
$$= \sum_{n=1}^{N_t} h(t-\tau_n : a_n), \qquad N_t \geq 1 \tag{9a}$$

In this expression, N_t is the number of raindrops that are encountered during the time $t-t_0$, where t_0 is the time of entry into the cloud, a_n the spherical radius, τ_n the time of impact for the nth drop, and $h(t-\tau_n : a_n)$ is a causal function which is the incremental contribution to F_t due to the nth impact.

Deterministic Excitation

A deterministic model was made for the purposes of increased understanding and for planning of experimental verification of the analytical predictions.

As a simplification to the multiple impact force model it is assumed that all drops have the same radius and that they encounter the structural model at a constant rate. The arrival rate is chosen by requiring that the mass encounter rate be the same as the mass encounter rate for a storm having a drop distribution given by Eq. (2). The arrival rate λ_D of the fixed size raindrops is

$$\lambda_D = 1/\Delta t = \dot{M}/M = \rho_L A_f V(4/3\pi\rho_w \bar{a}^3)^{-1} \tag{9b}$$

where $\bar{a}$ is the fixed drop radius. With these assumptions N_t, the number of drops encountered in a given interval of time, is fixed.

Statistical Excitation

For the statistical model, N_t is a stochastic process and a_n and τ_n are sequences of random variables. The raindrop sizes a_n are independent, identically distributed random variables with probability density given by Eq. (2). N_t is modeled as a Poisson process with intensity λ, as given by Eq. (4). The impact occurrence times are random variables whose statistical properties are determined from those of N_t.

Structural Model

As a simple model of rain-induced vibration of a space vehicle, we consider the purely axial response of an elementary rod of circular cross section excited at one end by the excitation F_t. The governing equations for the rod response are

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \tag{10}$$

and

$$AE\frac{\partial u}{\partial x}(0,t) = F_t \tag{11}$$

where $c = \sqrt{E/\rho}$ is the wave speed, u the axial displacement, x the axial coordinate, t the time, A the constant cross-sectional area of the rod, E the Young's modulus, and ρ the density of the material. The initial conditions are $u(x,0) = \partial u/\partial t(x,0) = 0$. The rod is semi-infinite and its response is meant to represent that in the nose of the space vehicle at an axial location at least several noise radii from the stagnation point. Obviously, this model omits many details of an actual space vehicle, but it can represent the axially propagating wave generated by a drop impact near the stagnation point.

The nose response is also modeled by the Mindlin-Herrmann rod theory which accounts for radial inertia and radial shear in addition to axial motion. The purpose of this is to assess the validity of the elementary theory in representing a raindrop-generated pulse, the length of which is short compared to a nose radius. The Mindlin-Herrmann equations[4] and end conditions are

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_D^2}\frac{\partial^2 u}{\partial t^2} = -\frac{2\lambda_l}{a_f(\lambda+2\mu)}\frac{\partial w}{\partial x} \tag{12}$$

$$\frac{\partial^2 w}{\partial x^2} - \frac{1}{c_s^2}\frac{\partial^2 w}{\partial t^2} = -\frac{4K_l^2\lambda_l}{a_f K^2\mu}\frac{\partial u}{\partial x} + \frac{8K_l^2(\lambda_l+\mu)}{a_f^2 K^2\mu}w \tag{13}$$

$$A_f(\lambda_l + 2\mu)\frac{\partial u}{\partial x}(0,t) + \frac{2A_f\lambda_l}{a_f}w(0,t) = F_t \tag{14}$$

$$\frac{\partial w}{\partial x}(0,t) = 0 \tag{15}$$

where u and x have the same meaning as in Eq. (10), w is the radial displacement at the rod surface, $c_D = \sqrt{(\lambda_l + 2\mu)/\rho}$ the dilatational wave speed, $c_s = \sqrt{K^2\mu/\rho}$ the shear wave speed, a_f the rod radius, K and K_l correction factors of the theory, λ_l and μ the Lame constants, and ν Poisson's ratio.

Analyses

Deterministic Model

The total force on the vehicle is represented as

$$F_t = \sum_{n=1}^{N} h(t - n\Delta T) \tag{16}$$

where $h(t)$ is given by Eq. (7), $N = N(t-t_0)/\Delta T$ is defined as the largest integer in $(t-t_0)/\Delta T$, and ΔT is the separation time between impacts. The time-averaged estimate of the mean value of the force is found as

$$\bar{F}(t) = \frac{1}{T}\int_0^T F_t \mathrm{d}t \tag{17}$$

and in the limit of large T is

$$\bar{F}(t) = I/\Delta T$$

Then the average force applied to the rod is simply interpreted as the product of the impulse due to the single fixed-size raindrop and the fixed rate of encounter of raindrops.

With I and ΔT determined by Eqs. (8) and (9b), the average force as predicted by the deterministic model is also given as

$$\bar{F} = k\rho_L V^2 A_f = 2P^* A_f \tag{18a}$$

where P^*, the so-called kinetic pressure, is

$$P^* = k\rho_L V^2/2 \tag{18b}$$

This formula shows that the time-averaged force is independent of the chosen raindrop size and that the only weather parameter upon which it is dependent is ρ_L, the liquid water content of the cloud system.

The time-averaged estimate of the force autocorrelation is expressed as

$$\langle F_t F_{t+\tau}\rangle = \left(\pi\frac{\bar{a}^2 P}{T}\right)^2 \int_0^T \sum_{n=0}^{N(T)} \sum_{m=0}^{N(T+\tau)}$$
$$\times \bar{h}(t - n\Delta T)\bar{h}(t+\tau - m\Delta T)\mathrm{d}t \tag{19}$$

where

$$\bar{h}(t) = \exp(-P\pi\bar{a}^2 t/I) \qquad t \geq 0$$
$$= 0 \qquad t \leq 0$$

which may be evaluated in the limit of large T to give

$$\hat{R}(\tau) = \langle F_t F_{t+\tau} \rangle = \frac{\pi\bar{a}^2 PI}{2\Delta T}\left(\frac{g_1(\tau)}{1-\bar{h}(\Delta T)} - \frac{g_2(\tau)}{1-\bar{h}(-(\Delta T))}\right) \tag{20}$$

where $g_1(\tau) = \bar{h}(\tau - N(\tau)\Delta T)$ and $g_2(\tau) = \bar{h}(N(\tau)\Delta T - \tau)$. It may be seen in Eq. (20) that the time-averaged autocorrelation is stationary, as would be anticipated. The functions $g_1(\tau)$ and $g_2(\tau)$ can also be expressed as

$$g_1(\tau) = \sum_{m=0}^{\infty} \bar{h}(\tau - m\Delta T) k(\tau, m) \tag{21}$$

and

$$g_2(\tau) = \sum_{m=0}^{\infty} \bar{h}(m\Delta T - \tau) k(\tau, m) \tag{22}$$

where

$$k(\tau, m) = H(\tau - m\Delta T) - H(\tau - (m+1)\Delta T)$$

where $H(t)$ is the Heaviside function.

What is not so readily apparent, but still might be anticipated from the nature of the excitation, is that the temporal estimate of the autocorrelation function is periodic with period ΔT.

The time-averaged estimate of the covariance is given as

$$\hat{C}(\tau) = \hat{R}(\tau) - \bar{F}^2 \tag{23}$$

Since $\hat{C}(\tau)$ is itself a periodic function with period ΔT it has a Fourier series representation as

$$\hat{C}(\tau) = \sum_{n=-\infty}^{\infty} C_n \exp(j2\pi n\tau/\Delta T) \tag{24}$$

where the Fourier coefficients were determined to be

$$C_n = 0 \qquad n = 0$$
$$= ((\Delta T/I)^2 + (2n/\bar{a}^2 P)^2)^{-1} \qquad n \neq 0 \tag{25}$$

From a knowledge of the covariance function the time-averaged estimate of the standard deviation of the excitation may be shown to be

$$\hat{\sigma} = \bar{F}(\pi\bar{a}^2 P\beta/(2\bar{F}) - 1)^{1/2} \tag{26a}$$

where

$$\beta = (1+e^{-\gamma})/(1-e^{-\gamma}) \text{ and } \gamma = p\pi\bar{a}^2/\bar{F} \tag{26b}$$

With the standard deviation and mean determined the coefficient of variation is written as

$$\hat{\sigma}/\bar{F} = (P\tilde{A}\beta/(4P^* A_f) - 1)^{1/2} \tag{26c}$$

where $\tilde{A}$ is given by

$$\tilde{A} = \pi\bar{a}^2 \tag{26d}$$

Statistical Model

To determine the statistical properties of the excitation it is convenient to determine the characteristic functional of the excitation defined as in Ref. 3 as

$$\phi_F(jv) = E\left[\exp\left(j\int_{t_0}^{T} F_\sigma \mathrm{d}v_\sigma\right)\right] \tag{27}$$

where E is the expectation operator, F_σ; $t_0 < \sigma \leq t$ the average force described by Eq. (9a), and v_σ an arbitrary real-value function subject to certain restrictions necessary to insure that the above integral converges in the mean square sense. With a proper choice of v_σ and operations on the characteristic functional, once it has been determined, the second-order statistics are readily obtained.

Following Ref. 3 the characteristic functional is obtained as

$$\phi_F(jv) = \exp\left\{\lambda\int_{t_0}^{T} E\left[\exp\left(j\int_{t_0}^{T} h(\sigma-\tau{:}a)\mathrm{d}v_\sigma - 1\right)\right]\mathrm{d}\tau\right\} \tag{28}$$

where the expectation within the integral is taken with respect to A, the random variable denoting the raindrop radius.

To evaluate the mean of the excitation v_σ is chosen as

$$v_\sigma = 0, \qquad t_0 \leq \sigma < t_1$$
$$= \alpha_1, \qquad t_1 \leq \sigma \leq t \tag{29}$$

Then the mean value of the excitation may be found from

$$E[F(t)] = \frac{1}{j}\frac{\partial \ln\phi_F(v)}{\partial\alpha_1}\bigg|_{\alpha_1=0} \tag{30}$$

to be

$$E[F(t)] = \lambda\int_{t_0}^{T} E[h(t-\tau{:}a)]\mathrm{d}\tau \tag{31}$$

If t_0 and T are considered to become very large negatively and positively, respectively, the mean value of the excitation can also be expressed as

$$E[F(t)] = \lambda E(I) \tag{32}$$

which has the simple interpretation that the average force is the product of the average raindrop encounter rate and the average impulse due to the encounter. The average excitation can also be expressed as

$$E[F(t)] = k\rho_L A_f V^2 \tag{33}$$

which is seen to be the same average force as predicted by the deterministic model.

The autocovariance function can be found from the characteristic functional by choosing the function v_σ in Eq. (27) to be

$$v_\sigma = 0, \qquad t_0 \leq \sigma < t_1$$
$$= \alpha_1, \qquad t_1 \leq \sigma < t_2$$
$$= \alpha_2, \qquad t_2 \leq \sigma < t$$

where α_1 and α_2 are constants. The autocovariance function is then obtained from the characteristic functional as

$$K_F(t_1, t_2) = -\frac{\partial^2 \ln\phi_F}{\partial\alpha_1\partial\alpha_2}\bigg|_{\alpha_1=\alpha_2=0} = E[F(t_1)F(t_2)] - E[F(t_1)]E[F(t_2)] \tag{34}$$

and after performing the differentiation the autocovariance is found to be

$$K_F(t_1, t_2) = \lambda\int_{t_0}^{T} E(h(t_1-\tau{:}a)h(t_2-\tau{:}a))\mathrm{d}\tau \tag{35}$$

where the expectation is to be taken with respect to the random variable A. In the limit for large T and $-t_0$ the autocovariance is

$$K_F(\beta) = \lambda \int_{-\infty}^{\infty} Eh(\tau{:}a)h[(\tau+\beta){:}a]\mathrm{d}\tau \tag{36}$$

which implies that after extended exposure the force excitation becomes stationary in the wide sense.

The covariance spectral density (CSD) of the excitation is found by taking the Fourier transform of Eq. (36) with respect to β to obtain

$$S_F(\omega) = \lambda E[\,|\hat{h}(\omega,a)|^2] = \lambda E[I^2/1 + (\omega I/\pi a^2 P)^2] \tag{37}$$

where ω is the circular frequency and $\hat{h}(\omega,a)$ the Fourier transform of the incremental force contribution $h(t)$. $S_F(\omega)$ can be evaluated by an asymptotic integration to find that

$$S_F(\omega) = 5!\,(E(A)\,V)^3 C_I[1 - 56(\omega/\omega')^2 + \ldots] \tag{38a}$$

where

$$C_I = 4/3\pi k^2 \rho_w \rho_L A_f \tag{38b}$$

and the characteristic frequency ω' is

$$\omega' = 3P\alpha/4k\rho_w V \tag{38c}$$

It is noted that the CSD increases as the third power of the average raindrop radius and the vehicle velocity. The variance of the excitation may be obtained from the CSD as

$$\sigma_f^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_F(\omega)\,\mathrm{d}\omega = \frac{\lambda}{2}E(\pi a^2 PI) \tag{39}$$

It may also be described by

$$\sigma_f^2 = K\frac{\rho_L v^2}{2}P\hat{A}A_f = P^* P\hat{A}A_f \tag{40a}$$

where $\hat{A}$ is given by

$$\hat{A} = 20\pi[E(A)]^2 \tag{40b}$$

Using the results of Eqs. (33) and (40) the coefficient of variation as predicted by the statistical model is given as

$$\sigma_f/\,|E(F)| = (P\hat{A}/P^* A_f)^{1/2}/2 \tag{41}$$

and the similarity with the deterministic model seen by comparing with Eq. (26c).

The covariance spectral density may also be written in nondimensional form as

$$\bar{S}_F(\Omega) = \alpha^3 S_F(\omega)/160\pi k^2 A_f V^3 \rho_w \rho_L \tag{42}$$

where the nondimensional frequency Ω is defined as

$$\Omega = (4k\rho_w V\omega)/3P\alpha$$

The covariance spectral density is evaluated numerically to obtain the graph of $\bar{S}_F$ vs Ω shown in Fig. 1. This curve defines a general relationship between all parameters of the rain vibration model.

Vibration Response in the Nose

The axial acceleration covariance spectral density in the nose of the space vehicle may be estimated from the elementary rod equations (10) and (11) and the force covariance spectral density equation (37). Using the Fourier

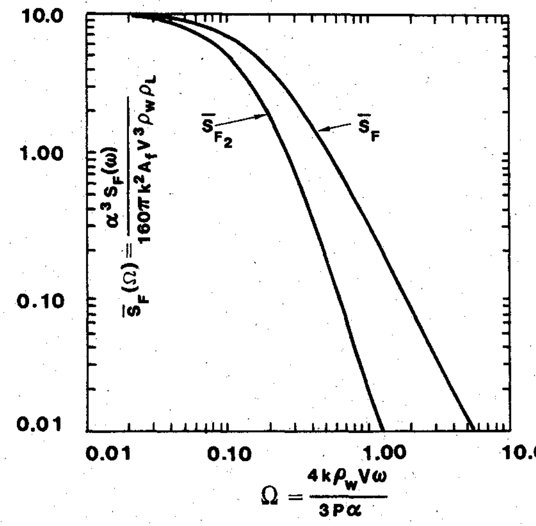

Fig. 1 Force covariance spectral densities for particle impact models.

transform with transform variable ω the squared magnitude of the transfer function $H(\omega)$ relating the acceleration at a point in the rod to a force applied at $x=0$ is

$$|H(\omega)|^2 = [\omega/\rho c A_f]^2 \tag{43}$$

then the acceleration covariance spectral density $S_A(\omega)$ is

$$S_A(\omega) = S_F(\omega)\,|H(\omega)|^2 \tag{44}$$

Equation (44) may be written in nondimensional form as

$$\bar{S}_A(\Omega) = \frac{\rho_w \rho^2 c^2 A_f \alpha}{90\pi P^2 V \rho_L} = \Omega^2 S_F(\Omega) \tag{45}$$

to obtain $\bar{S}_A$ vs ω as shown in Fig. 2.

The acceleration response variance which is associated with the area under the S_A curve of Fig. 2, is unbounded. This is due to two causes: 1) the structural model is undamped and 2) the form of the force model. Although the force model itself has a finite variance, when it is used in conjunction with the simple undamped rod model an unbounded acceleration response variance is obtained. To obtain a meaningful result a single-particle excitation model with a more realistic finite rise time is used. This model is given by

$$h(t) = h_0^2 t/I\exp(-h_0 t/I) \qquad t \geq 0$$

$$= 0 \qquad t < 0 \tag{46}$$

where the maximum value is $h_0 e^{-1}$ and the total impulse is I.

The CSD associated with this pulse is obtained from Eq. (37) as

$$S_{F_2}(\omega) = E[I^2/(1 + (\pi I/\omega a^2 p)^2)]^2 \tag{47}$$

The expectation with respect to the random variable A is evaluated numerically to obtain the nondimensional CSD vs nondimensional frequency curve shown in Fig. 1. The estimate of the acceleration covariance spectral density of a point in the nose of a space vehicle is obtained from Eq. (45) with the nondimensional excitation $\bar{S}_{F_2}(\Omega)$ substituted for $\bar{S}_F(\Omega)$. The nondimensional covariance spectral density of the response $\bar{S}_{A_2}(\Omega)$ is plotted vs Ω and shown in Fig. 2. The nondimensional variance of the response which is associated with the area under the curve is found to be

$$\bar{\sigma}_{A_2}^2 = 7/4 \tag{48}$$

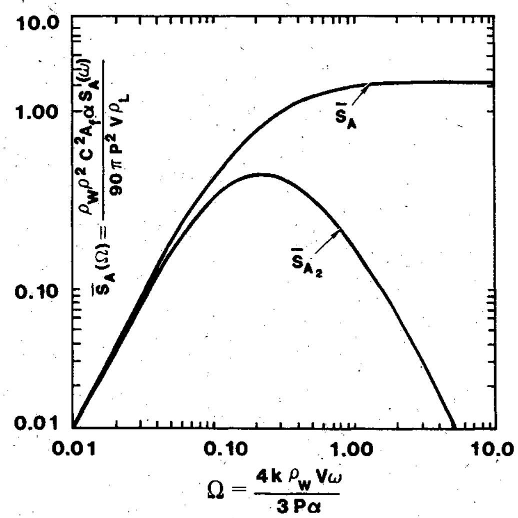

Fig. 2 Acceleration response covariance spectral densities.

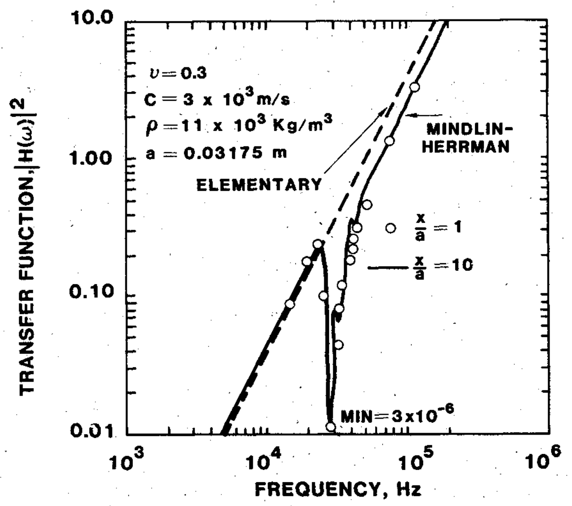

Fig. 3 Comparison of elementary rod theory and Mindlin-Herrmann theory.

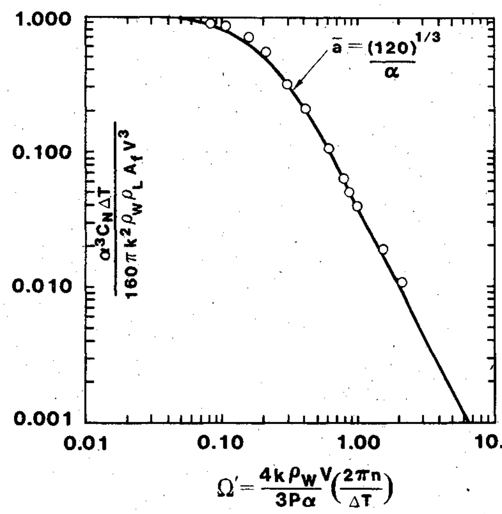

Fig. 4 Faired nondimensional Fourier coefficients of deterministic model.

The results of Fig. 2 and Eq. (48) can be used either as an estimate of the acceleration response of rigidly mounted components located in the nose of a space vehicle or as input to a separate component response analysis.

In Fig. 2 the response $\bar{S}_{A_2}$ increases with frequency to peaks which occur at frequencies of 0.1-10 MHz for typical values of the physical parameters. To assess the validity of elementary rod theory to represent high-frequency waves, a comparison is made in Fig. 3 of the squared magnitudes of the transfer functions from the elementary theory and the Mindlin-Herrmann theory ($K^2 = 0.869$ and $K_I = 0.477$ corresponding to $\nu = 0.3$ were used[4]). The main difference is the dip in $|H(\omega)|^2$ from the Mindlin-Herrmann theory at a frequency of about 3×10^4 Hz which corresponds to a purely radial mode of rod motion. Elsewhere the agreement is good with a difference of about 20% in the high-frequency asymptotes. From these results one might expect more exact rod theories to give additional dips in $|H(\omega)|^2$ at higher frequencies. Since the elementary theory allows no radial motion, we might expect it to overestimate $|H(\omega)|^2$ from more exact rod theories as indicated in the particular case in Fig. 3. This conclusion may not hold for the response of a particular point (recall $|H(\omega)|^2$ represents the average cross-sectional response) which could vary significantly with radial location when the response is dominated by high-frequency components, as suggested by Davies.[5]

Comparison and Results of the Two Force Models

As the deterministic model is intended for testing simulation of the effects due to the more realistic statistical model it is desirable to make the two models agree as well as possible. It has already been seen that the two models predict the same average force. To obtain further agreement the fixed-size raindrop of the deterministic model was chosen such that a faired curve of the nondimensionalized Fourier coefficients of the time-averaged covariance from the deterministic model vs nondimensional frequency would exactly match the low-frequency portion of the CSD curve of the statistical model. The faired curve from the deterministic model which approximates the CSD is shown in Fig. 4 and may be compared with the CSD of the statistical model from Fig. 1. The fixed-size raindrop radius chosen to achieve this objective was found to be

$$\bar{a} = (120)^{1/3} E(A) \tag{49}$$

With a so chosen it is seen that the coefficient of variation [Eq. (26c)] for the deterministic model, for typical values of the physical parameters, is approximated very well by

$$\frac{\hat{\sigma}}{|\bar{F}|} = \frac{1}{2}\left(\frac{P\tilde{A}}{P^* A_f}\right)^{1/2} \tag{50}$$

A comparison of this result with the coefficient of variation for the statistical model [Eq. (41)] indicates the excellent agreement of the two models.

The average force due to the rain as compared to that force due to the atmosphere may be shown to be

$$F_A/\bar{F} = \rho_A/2k\rho_L \tag{51}$$

where F_A and $\bar{F}$ are average forces due to the atmosphere and rain, respectively, and ρ_A the mass density of the atmosphere at the particular altitude at which the vehicle is traveling. The minimum of this ratio is of the order of 10; thus, it is apparent that the average force due to the weather encounter is small compared to that due to the atmosphere.

Conclusions

Two models, one statistical, the other deterministic, describing the force F_t on the nose of a space vehicle traveling at constant velocity through a rain cloud have been formulated. The deterministic model, which is intended for testing purposes, is made to agree with the more realistic statistical model by choosing its fixed-size raindrops such that the approximated covariance spectral density of the deterministic model agrees, in the low-frequency regime, with the statistical model. The two models predict the same average force and show excellent agreement in their coefficients of variation and covariance spectral densities. The covariance spectral density of the excitation is nearly flat at low frequencies and is down 10 dB at frequencies near 100 kHz for practical values of the physical parameters. The average force due to the weather encounter is shown to be small compared to that due to the atmosphere. The coefficient of variation as predicted by the models is of the order of 20 for typical values of the parameters.

The rain-induced vibration on a space vehicle traveling through a rain cloud is modeled in two ways by considering the axial response of an elementary rod and a Mindlin-Herrmann rod excited at one end by the force F_t. A comparison suggests that the elementary response theory provides an upper bound on the average cross-sectional response in the axial direction. The maximum response from the elementary theory occurs at frequencies on the order of 1 MHz.

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References

[1] Marshall, J. S. and Palmer, W. McK., "The Distribution of Raindrops with Size," *Journal of Meteorology*, 1948, Vol. 80, pp. 165-166.

[2] Strike, R. and Lasker, G., "Contact Fuze Study—Final Technical Report," Vols. 1 and 2, General Dynamics Pomona Div., Rept. SAMSO-TR-77-88.

[3] Snyder, D. L., *Random Point Processes*, John Wiley & Sons, New York, 1975.

[4] Mindlin, R. D. and Herrmann, G., "A One-Dimensional Theory of Compressible Waves in an Elastic Rod," *Proceedings of First U.S. National Congress of Applied Mechanics*, June 1951, ASME, New York, 1952, pp. 187-191.

[5] Davies, R. M., "A Critical Study of the Hopkinson Pressure Bar," *Royal Society of London Philosophical Transactions*, Ser. A, Vol. 240, Jan. 1948, pp. 375-457.